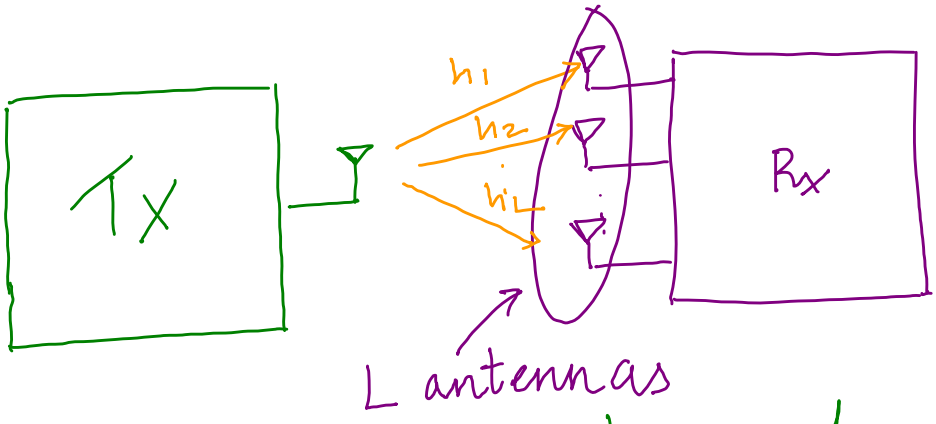


$h_1, h_2, \ldots, h_L$ — channel coefficients

Optimal Receive combining is Maximal Ratio Combiner

$$SNR_m = \frac{P}{\sigma^2}\left\{\|h\|^2\right\}$$

$$= \frac{P}{\sigma^2}\left\{|h_1|^2 + |h_2|^2 + \cdots + |h_L|^2\right\}$$

SNR after MRC

$$g = \|h\|^2$$

$$= |h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$$

BER of Multiple Antenna Wireless System:

$$g = |h_1|^2 + |h_2|^2 + \cdots + |h_L|^2$$

$h_1, h_2, \ldots, h_L$ — Independent Identically distributed (IID)

- Rayleigh Fading Channel coefficients
- $E\{|h_i|^2\} = 1$

$$g = \|Th\|^2$$

Chi-Squared Random variable

$$\chi^2_{2L}$$

2L Degrees of Freedom

$$F_G(g) = \frac{1}{(L-1)!} g^{L-1} e^{-g}$$

Received SNR,

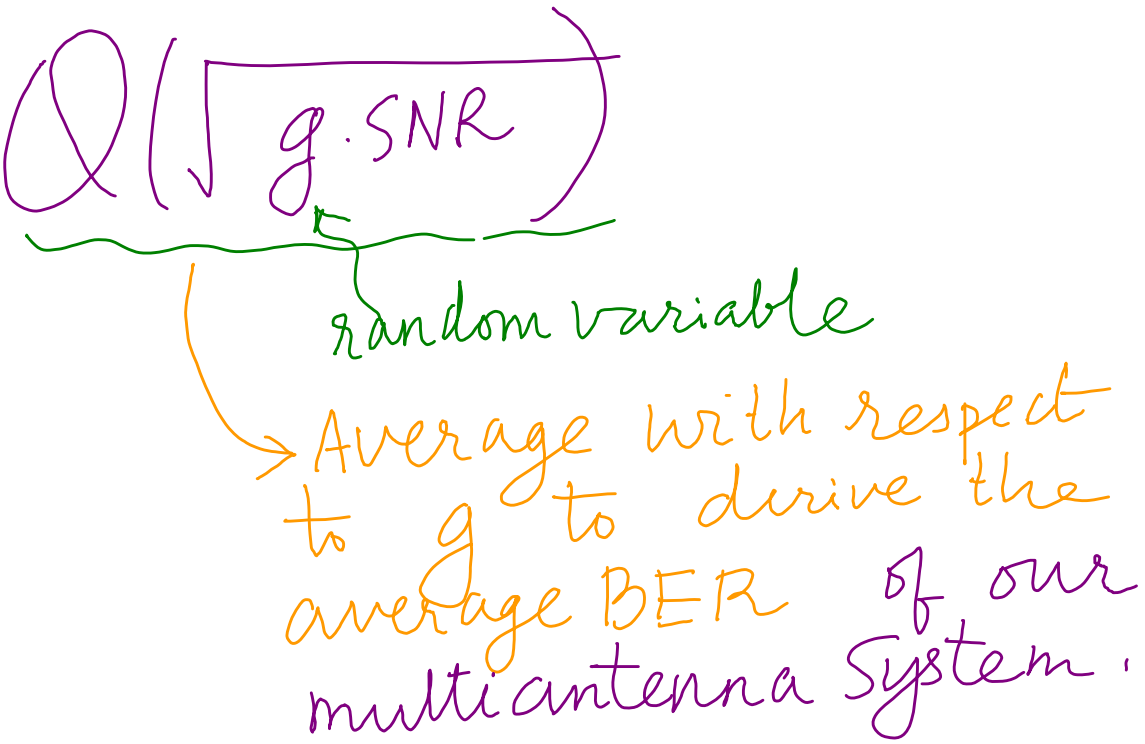
$$SNR_m = \|h\|^2 \frac{P}{\sigma^2} = g \frac{P}{\sigma^2}$$

$$= g \cdot SNR$$

From previous BER Analysis

$$BER = Q\left(\sqrt{SNR_m}\right)$$

$$= Q\left(\sqrt{g\, SNR}\right)$$


$Q(\sqrt{g \cdot SNR})$
random variable
Average with respect to g to derive the average BER of our multicantenna System.

Average BER

$$= \int_0^\infty Q\left(\sqrt{g\,SNR}\right) F_G(g)\, dg$$

$$= \int_0^\infty Q\left(\sqrt{g\,SNR}\right) \cdot \frac{g^{L-1} e^{-g}}{(L-1)!}\, dg$$

can be simplified as,

$$= \frac{(1-\lambda)^L}{2} \cdot \sum_{l=0}^{L-1} {}^{L+l-1}C_l \left(\frac{1+\lambda}{2}\right)^l$$

where $\lambda = \sqrt{\frac{SNR}{2+SNR}}$

Average BER of a multiple antenna system

$$= \left(\frac{1-\lambda}{2}\right)^{L} \cdot \sum_{l=0}^{L-1} {}^{L+l-1}C_{l} \cdot \left(\frac{1+\lambda}{2}\right)^{l}$$

$${}^{n}C_{k} = \frac{n!}{k!\,(n-k)!}$$

For $L = 2$

$$BER = \left(\frac{1-\lambda}{2}\right)^2 \sum_{l=0}^{1} {}^{l+1}C_l \left(\frac{1+\lambda}{2}\right)^l$$

$$= \left(\frac{1-\lambda}{2}\right)^2 \left\{ {}^{1}C_0 \left(\frac{1+\lambda}{2}\right)^0 + {}^{2}C_1 \left(\frac{1+\lambda}{2}\right)^1 \right\}$$

$${}^{1}C_0 = \frac{1!}{0!\ 1!} = 1$$

$${}^{2}C_1 = \frac{2!}{1!\ 1!} = 2$$

$$= \left(\frac{1-\lambda}{2}\right)^2 \left\{1 + 2\left(\frac{1+\lambda}{2}\right)\right\}$$

$$= \left(\frac{1-\lambda}{2}\right)^2 \left\{2 + \lambda\right\}$$

Average BER with $L = 2$ antennas

$$\lambda = \sqrt{\frac{SNR}{2 + SNR}}$$

Example: What is BER of a system with $L = 2$ antennas at $SNR_{dB} = 20$ dB?

$$SNR_{dB} = 10 \log_{10} SNR = 20$$

$$SNR = 10^2 = 100.$$

$$\lambda = \sqrt{\frac{SNR}{2 + SNR}} = \sqrt{\frac{100}{102}} = 0.9901$$

Average BER for $L = 2$

$$= \frac{(1-\lambda)^2}{2} \cdot (2+\lambda)$$

Substituting $\lambda = 0.9901$

Average BER $= 7.2564 \times 10^{-5}$